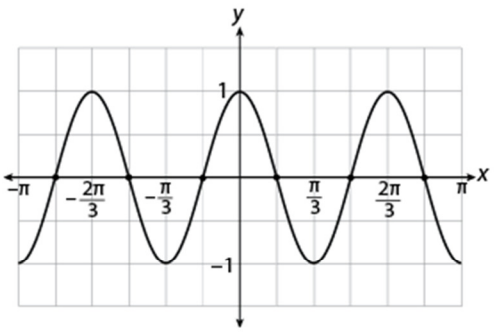
Advanced Geometry, Trigonometry Notes **Solutions**

1.



The graph of $g(x) = \cos(3x)$ is shown above. Which of the following lists represents the values of x for which $g(x) = 0$?

Ⓐ $-180°, -120°, -60°, 60°, 120°, 180°$
Ⓑ $-165°, -105°, -45°, 45°, 105°, 165°$
Ⓒ $-150°, -90°, -30°, 30°, 90°, 120°$
Ⓓ $-120°, -80°, -40°, 40°, 80°, 120°$

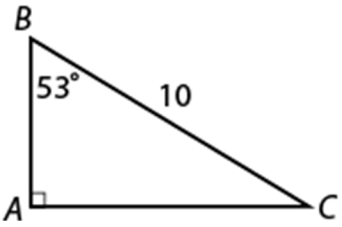
Difficulty: Hard

Category: Additional Topics in Math / Trigonometry

Strategic Advice: You don't need to know a lot of trig to answer this question. You will, however, need to know how to convert radians to degrees (multiply the radians by $\frac{180°}{\pi}$).

Getting to the Answer: Recall that $g(x) = 0$ means "crosses the x-axis," regardless of the type of function involved, which means you are looking for the x-intercepts. Study the graph carefully: The function crosses the x-axis six times, halfway between each of the labeled grid-lines. Rather than finding the points using the radians given in the graph, convert the radians to degrees and then determine the halfway points: $-\pi\left(\frac{180°}{\pi}\right) = -180°$, $\left(-\frac{2\pi}{3}\right)\left(\frac{180°}{\pi}\right) = -120°$, $\left(-\frac{\pi}{3}\right)\left(\frac{180°}{\pi}\right) = -60°$, and so on. Take a minute now to find the halfway points because chances are that you don't have to do all the conversions. Halfway between $-180°$ and $-120°$ is $-150°$. Stop—that's all you need to know. The leftmost x-intercept is at $-150°$, which means (C) must be correct. If you want to check another value just to be sure, halfway between $-120°$ and $-60°$ is $-90°$, which is the second value in (C), confirming that it is correct.

2.



Based on the figure above, what is the approximate length of side AB?

Ⓐ 6
Ⓑ 7.2
Ⓒ 8
Ⓓ 8.5

Difficulty: Medium

Category: Additional Topics in Math / Trigonometry

Strategic Advice: *ABC* is a right triangle. You know the length of one side and the measure of one of the acute angles, which means you can use SOH CAH TOA.

Getting to the Answer: You know the length of the hypotenuse (10) and you're looking for the length of the side adjacent to (touching) the 53° angle, so use cosine. Set up a trigonometric ratio and solve for the length of *AB*. Make sure your calculator is set to degree mode.

$$\cos(53°) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos(53°) = \frac{AB}{10}$$

$$10(0.601815) = AC$$

$$6.01815 = AC$$

The length of *AB* is approximately 6.

3.

In geology, the water table is the level below which the ground is saturated with water. Wells must be dug below this point to bring water up into the well. Except in cases of severe flooding, the water level in a well does not rise above the water table. Suppose a cylindrical well is 6 feet wide and 60 feet deep in an area where the water table is 40 feet below ground level. Assuming no unusual circumstances, what is the volume in cubic feet of the water in the well at any given time?

Ⓐ 180π
Ⓑ 360π
Ⓒ 540π
Ⓓ 720π

Difficulty: Hard

Category: Additional Topics in Math / Geometry

Strategic Advice: Use the formula for finding the volume of a cylinder, $V = \pi r^2 h$.Check the formula page rather than trying to recall it from memory. Don't forget to use the radius, not the diameter, which is given.

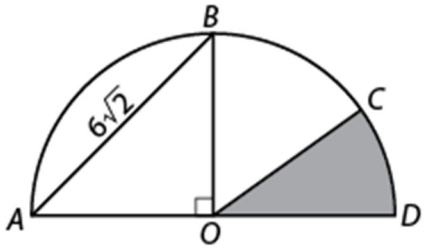
Getting to the Answer: The well is 6 feet wide; this is its diameter, so $r = 3$. The height of the well is 60 feet, but the water table is 40 feet below ground level, which means only 60 – 40 = 20 feet of the well is below the water table and thus has water in it, so $h = 20$.

$V = \pi(3)^2(20)$

$V = \pi(9)(20)$

$V = 180\pi$

4.



If segment *AD* is a diameter of the circle shown above, and the length of arc *CD* is π, what is the area of the shaded region? Use 3.14 to approximate π and round your answer to the nearest tenth.

Difficulty: Hard

Category: Additional Topics in Math / Geometry

Strategic Advice: This question requires logical thinking, knowledge of special right triangles, and knowing how to find arc length. It's a challenging question, so if you're pushed for time, skip it and come back later.

Getting to the Answer: Finding the area of a sector of a circle (the shaded region) requires knowing the degree measure of the corresponding interior angle. Given that information, your first step is to find the area of the entire circle. Then you'll find the proportional amount represented by the sector. To find the area of a circle, the only thing you need is the radius. The radius is not shown in the figure, so you will have to think about special right triangles. In the figure, triangle *ABO* is formed by 2 radii and a 90° angle. This means the triangle must be a 45-45-90 triangle, and therefore its side lengths are in the ratio $1:1:\sqrt{2}$. The hypotenuse is given as $6\sqrt{2}$, so the side lengths of the triangle, and therefore the radius of the circle, must be 6, and the area of the entire circle is $A = \pi r^2 = \pi(6) = 36\pi$. Now you need to find the portion of the circle represented by the shaded region by finding the measure of the angle inside the sector and dividing by 360. You'll need to use the given arc length, π, and the formula for finding arc length (arc length = θr, where θ is the interior angle and r is the length of the radius):

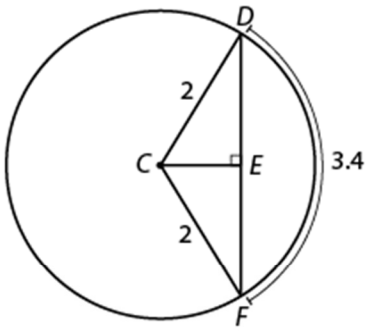
$\pi = \theta(6)$

$\frac{\pi}{6} = \theta$

If you know your unit circle, you know this corresponds to 30°. If you don't recall this fact, then you can convert radians to degrees by multiplying the radian measure by $\frac{180}{\pi}$ to get $\frac{\pi}{6} \times \frac{180}{\pi} = \frac{180}{6} = 30$.

This means the shaded region makes up $\frac{30}{360} = \frac{1}{12}$ of the total area of the circle, so divide the total area by 12 to get $36\pi \div 12 = 3\pi$. The question tells you to approximate π using 3.14 and to round to the nearest tenth, so the final answer is 9.4.

5. Calculator



Which of the following gives the length of chord *DF* in the figure above?

Ⓐ 2cos(1.7)
Ⓑ 2sin(1.7)
Ⓒ 4cos(0.85)
Ⓓ 4sin(0.85)

Difficulty: Hard

Category: Additional Topics in Math / Trigonometry

Strategic Advice: This is a very difficult question involving arc length given in radians and answer choices that involve trig functions. If you're not familiar with these topics, you should guess and move on to the next question. (Don't forget—there is no penalty for wrong answers on the new SAT!)

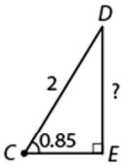
Getting to the Answer: Take a peek at the answer choices—the angles of the trig functions are given in radians, rather than degrees (you know this because there is no degree symbol). This means you'll need to use the radian formula for finding arc length: $arcL = \theta \times r$, where θ is the central angle of the arc in radians and r is the radius of the circle. This will allow you to determine the measure of the central angle, half of which becomes one of the angles of a right triangle (*CDE*, for example). You know both the arc length (3.4) and the radius (2), so solve for the central angle.

$arcL = \theta \times r$

$3.4 = \theta \times 2$

$1.7 = \theta$

This means that angle *DCF* has a measure of 1.7 radians, and consequently, angle *DCE* has a measure of half that, or 0.85 radians. Add this measure to the triangle, or draw a quick right triangle off to the side like the one below:

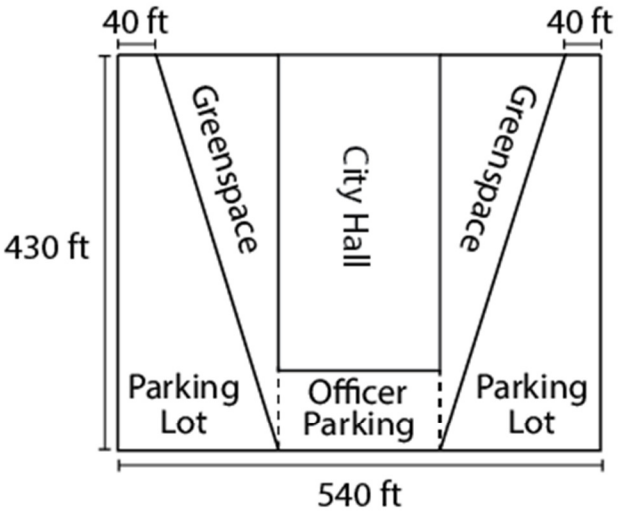


Now, if you can find the length of side *DE*, you can double it to find the length of chord *DF*. Side *DE* is opposite the angle measure that you found and you know the hypotenuse of the triangle; so, use the ratio $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$ to find the length of side *DE*:

$$\sin(0.85) = \frac{DE}{2}$$
$$2\sin(0.85) = DE$$

Multiply by 2 to find that $DF = 2 \times 2\sin(0.85) = 4\sin(0.85)$. Keep in mind that multiplying the angle (inside the parentheses) is not the same as multiplying the whole quantity by 2.

6. Calculator



Many cities try to work "greenspaces" into their city planning because living plants help filter the city's air, reducing the effects of pollution. The figure above shows the plans for a new greenspace around City Hall, which will be created by converting portions of the existing parking lots. If the width of each parking lot is the same as the width of the City Hall building, how many thousands of square feet of greenspace will there be after the conversion? Round to the nearest thousand and enter your answer in terms of thousands. (For example, enter 14,000 as 14.)

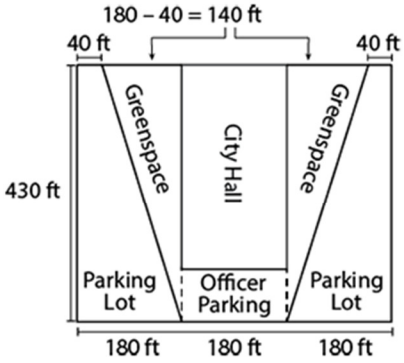
Difficulty: Medium

Category: Additional Topics in Math / Geometry

Strategic Advice: Whenever a question asks about the amount of space something covers (here, the greenspace), you are looking for area. In this question, the area that you're looking for takes on the shape of a right triangle (actually, two of them), so use the formula $A = \frac{1}{2}bh$.

Getting to the Answer: The key to answering this question is in labeling the diagram. The calculations are very straightforward once you have the correct dimensions of the triangles. You're given that the width of each parking lot is equal to the width of the City Hall building, so each parking lot is 540 ÷ 3 = 180 feet wide. This means the base of each triangle (at the top of the diagram) is 180 – 40 = 140 feet.

The height of each triangle is the same as the length of the parking lot, which is 430 feet.



You now have all the numbers you need. The area of each triangle is $\frac{1}{2}(140)(430) = 30{,}100$, so both triangles together result in a greenspace that covers 60,200 square feet. Rounded to the nearest thousand, this is 60,000, which should be gridded in as 60.

7.

If x is an angle such that $0 < x < 90°$, which of the following statements is not always true?

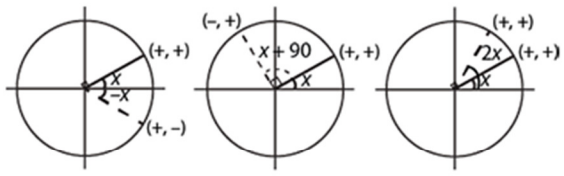
Ⓐ $\cos(x) > 0$
Ⓑ $\cos(-x) > 0$
Ⓒ $\cos(x + 90°) < 0$
Ⓓ $\cos(2x) < 0$

Difficulty: Hard

Category: Additional Topics in Math / Trigonometry

Strategic Advice: When a trig question involves signs (+ and −) rather than numbers, using your knowledge of how the *unit circle* works will get you to the answer. Before you begin analyzing the answer choices, translate the inequality symbols to signs (> 0 means +, and < 0 means −).

Getting to the Answer: Draw a quick sketch of a unit circle and add the angle x. Because $0 < x < 90°$, you can draw the angle anywhere in the first quadrant. Drawing a small angle is usually the safest way to go. Then, sketch in each of the angles described in the answers. Keep in mind that the cosine of the angle is equal to the x-coordinate of the point where the angle hits the unit circle, so it's positive in Quadrants I and IV, and negative in Quadrants II and III.



After sketching in each of the angles, compare them with the answer choices. The only one that is not always true is (D). When the angle is small, twice the angle still lands in Quadrant I, so the cosine is still positive (and therefore > 0, not < 0). Note that had you drawn a larger angle in Quadrant I, such as a 60°-degree angle, $2x$ would have landed in Quadrant II, in which case the cosine would have been negative. However, the question asks for the statement that is not *always* true, so you must consider both large and small angles in Quadrant I.